Lecture 14. **Problem definition of synthesis of ACS. Algorithm of the solving of the task of parametric synthesis of the linear ACS. Example**

When projecting a control system for any CO, it is surely necessary to make an *accurate, speeding system with great stability factor*. In this case, first of all, they try rationally and in permissible limits to change the parameters (coefficients of separate links transfer, time constants, etc.) in such a way, in order to satisfy to specified requirements of quality controlling, which are characterized by quality criteria. When it is impossible to solve this task within the existing system we have to change its structure. For this purpose we usually use introduction of corrective means into the system – these means must change the dynamics of the whole system in required direction. Corrective links are the links with definite transfer functions.

14.1 Problem definition parametric synthesis of ACS

Verbal formulation of the task is the following: we have a controlled object, principle of control – on mismatch is specified and the system is covered by proportional negative Feedback. It is required to characterize a corrective link at CO specified characteristics, sourcing from the requirements, made to the system.

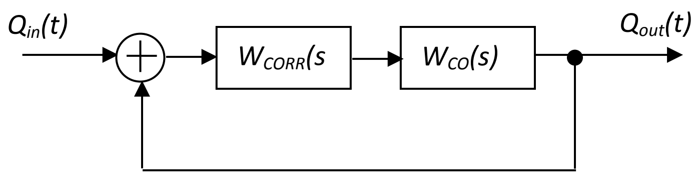

Fig. 5.2b. Closed-loop system with a corrective device

Requirements:
1) to the transfer function *h(t)* (transient process quality);
2) to an error in steady-state regime.

Mathematical setting of the task

Let specified the following:
– structure of the system, i.e. principle of control;
– LgGFC of controlled object (CO) $L_{CO}(\omega) = \ln|W_{CO}(j\omega)|$;
– desired LgGFC $L_*(\omega) = \ln|W_*(j\omega)|$, originating from requirements to transient process.
It is required (necessary) :
– to select a transfer function of corrective device $W_{corr}(s)$;
– to define the parameters $W_{corr}(s)$.
At any limits and relevant requirements to quality indexes $0 < K_i < K_{\lim it}$ $\forall i = \overline{1,n}$; $0 < T_i < T_{i\ assume}$ $\forall i = \overline{1,n}$; $t_w \le t_{assume}$; $\delta \le \delta_{assume}$.

Algorithm and solution

1. We will write transfer function of closed system.

Frequency characteristics of a closed-loop system will be written in the following way:

$$W_{closed}(j\omega)=\frac{W_{CORR}(j\omega)W_{CO}(j\omega)}{1+W_{CORR}(j\omega)W_{CO}(j\omega)}. \quad (5.1)$$

2. Now let's write the transfer function of a closed-loop system $W_{cl}(j\omega)$ as the desired transfer function. The desired one $W_*(j\omega)$ is characterized by properties of an open-loop system:

$$W_{closed}(j\omega)=\frac{W_*(j\omega)}{1+W_*(j\omega)} \quad (5.2)$$

3. We will find transfer function of the corrective link.

If the left parts of (5.1) and (5.2) are equal, then the right parts of (5.1) and (5.2) of equations are equal as well. Consequently,

$$W_*(j\omega)=W_{CORR}(j\omega)W_{CO}(j\omega).$$

Hence, $W_{CORR}(j\omega)=\dfrac{W_*(j\omega)}{W_{OC}(j\omega)}$.

4. We will write of LgGFC f the correcting device.

Having taken the logarithm on the last equation, we'll get LgGFC of corrective device:

$$\ln|W_{CORR}(j\omega)| = \ln|W_*(j\omega)| - \ln|W_{CO}(j\omega)|. \quad (5.3)$$

5. We will define the rule of creation of approximating LgGFC of the corrective link.

As approximating LgGFC is made by segments with inclines *± 20 [db/dec]*, then we should subtract by segments of tracking frequencies. So, we have obtained the rule of LgGFC characterization of corrective device.

Rule: to obtain approximating LgGFC of corrective device, it is necessary to subtract from desired LgGFC the LgGFC of CO original, subtraction must be carried out by segments up to tracking frequencies.

By what is the incline *± 20 [db/dec]* characterized? By introducing differential and integral links into the system. Let's analyze the following example.

Example 5.1. Let LgGFC of CO is given, presented in Fig. 5.3.

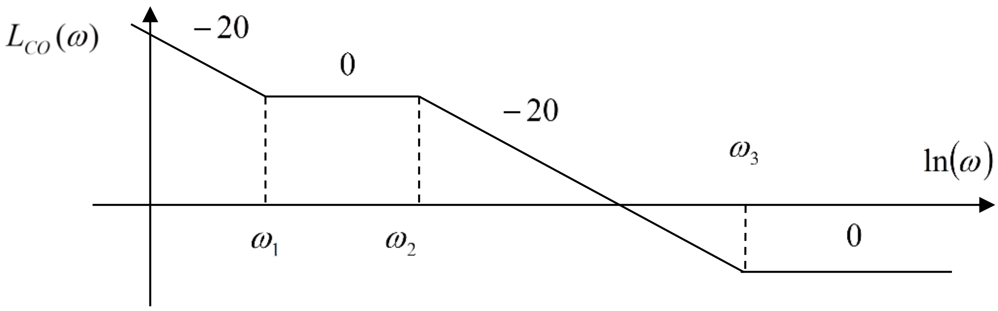

Fig. 5.3. LgGFC of control object

If LgGFC crosses the axis of frequencies with incline - 20 [db/dec], then stability of the system is guaranteed; if it is - 40 [db/dec], then the system may be oscillating, it is necessary to check the following: if it is not at oscillation limit?

Desired LgGFC has the following, presented on pic. 5.4.

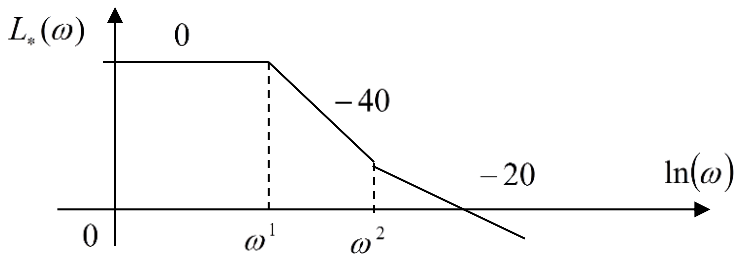

Fig. 5.4. Desired LgGFC

Using the rule (5.3), by which in order to get approximating LgGFC of corrective link, it is necessary from desired LgGFC to subtract LgGFC of original CO; it should be subtracted by segments up to tracking frequencies, we'll obtain LgGFC of corrective link.

6. We will construct approximating LgGFC of the corrective link.

Let's make both LgGFC $\ln|W_*(j\omega)|$ and $\ln|W(j\omega)|$ in one graph (Fig. 5.5); by the rule (5.3) we'll get LgGFC of corrective link, presented at Fig. 5.5:

$$\ln|W_{CORR}(j\omega)| = \ln|W_*(j\omega)| - \ln|W_{CO}(j\omega)|.$$

7. By the form (in parts) approximating LgGFC of the corrective link we will write its transfer function.

Now we'll write the transfer function of corrective link:

$$W_{CORR}(s)=\frac{\tau'_0\,s\cdot 1\left(\tau'''^2 s^2+2\xi\tau''s+1\right)}{(\tau's+1)\left(\tau''^2+s^2+2\xi\tau''s+1\right)\left(\tau^{iv}s+1\right)},$$

where $\tau'_0=\frac{1}{\omega'_0}$ (+20 db/dec, ω'_0 is a tracking frequency of differential link);

$\tau'=\frac{1}{\omega'}$ (0 db/dec, ω' is a tracking frequency of aperiodic link of the first order, was +20-20=0 db/dec);

$\tau''=\frac{1}{\omega''}$ (-40 db/dec, ω'' is a tracking frequency, that's why aperiodic link of the second order);

$\tau'''=\frac{1}{\omega'''}$ (0 db/dec, ω''' is a tracking frequency, that's why aperiodic link of the second order: was –40+40 =0 db/dec);

$\tau^{IV}=\frac{1}{\omega^{IV}}$ (-20 db/dec, ω^{IV} is a tracking frequency, that's why aperiodic link of the first order).

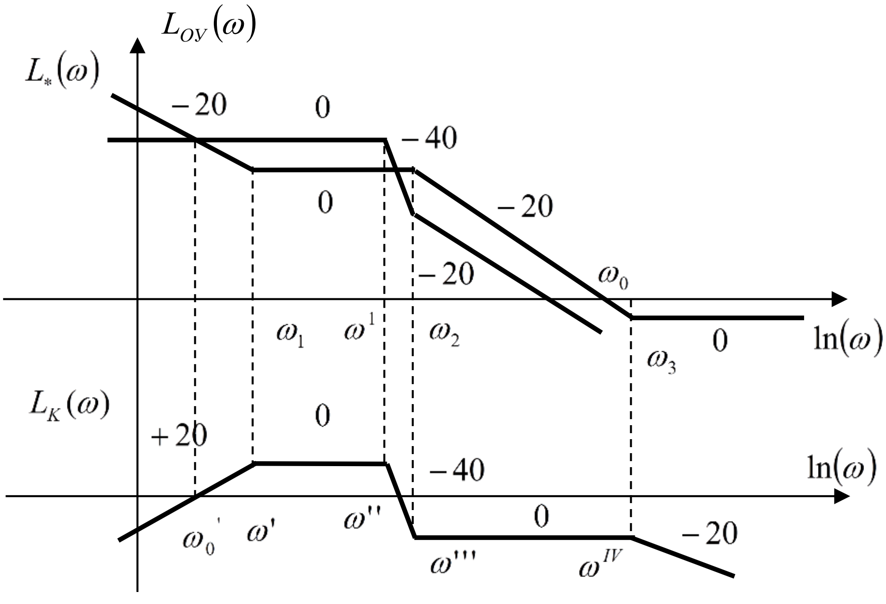

Fig. 5.5. LgGFC of corrective device

8. Implementation of the correcting device on the found its transfer function.

Let's view a realization process of the corrective device on an example of electric filters (passive electric 4 contact pole) (Fig. 5.6).

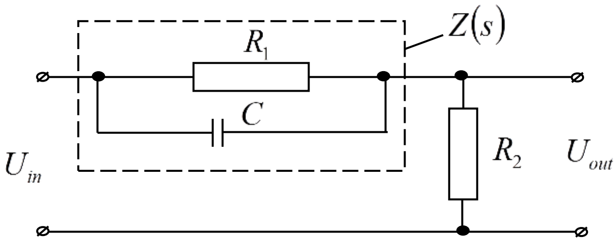

Fig. 5.6. Scheme of a corrective link

$$Z(s)=\frac{R_1\frac{1}{cs}}{R_1+\frac{1}{cs}}=\frac{R_1}{R_1cs+1}.$$

$$W_K(s)\overset{\Delta}{=}\frac{U_{вых}(s)}{U_{вх}(s)}=\frac{R_2}{R_2+Z(s)}=\frac{R_2}{R_2+\frac{R_1\frac{1}{cs}}{R_1+\frac{1}{cs}}}=\frac{R_2(R_1cs+1)}{R_2(R_1cs+1)+R_1}=$$

$$=\frac{R_2R_1cs+1}{R_2R_1cs+(R_2+R_1)}=\frac{K_1}{K_2}\cdot\frac{T_2's+1}{T_2''s+1}=K\frac{1+\tau_1 s}{1+\tau_2 s}.$$

a) If $\tau_1>\tau_2$, then $\omega_1>\omega_2$; $1<K<10$

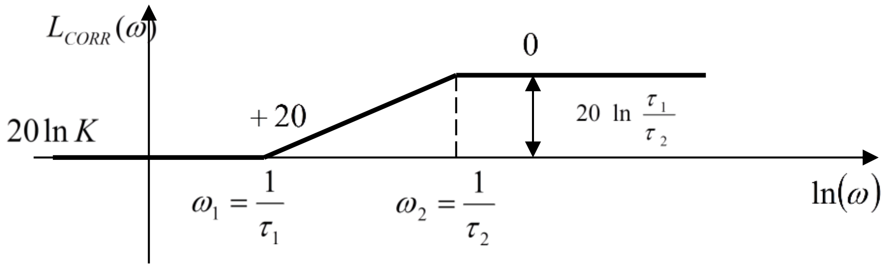

Fig. 5.7a. LgGFC of corrective device *a)*

This corrective path can be called proportionally-differential.

b) If $\tau_1<\tau_2$, then $\omega_1<\omega_2$; $1<K<10$.

This corrective path can be called proportionally-integrating.

Various types of corrective devices are described in V.A. Besekerskiy’s (В.А. Бесекерский) tables [3].

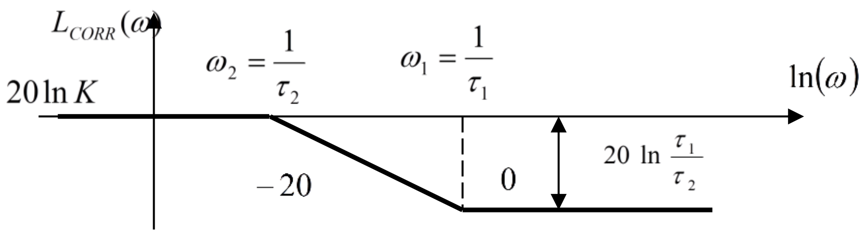

Fig. 5.7b. LgGFC of corrective device *b)*

14.2 Building the desired LgGFC

During the synthesis of a corrective device it is necessary to build the desired LgGFC. The desired LgGFC is called asymptotic LgGFC $L_*(\omega)$ of an open-loop system, which has desired (required) static and dynamic properties. Desired LgGFC (Fig. 5.8) consists of three main asymptotes: low-frequency, mid-frequency and high-frequency.

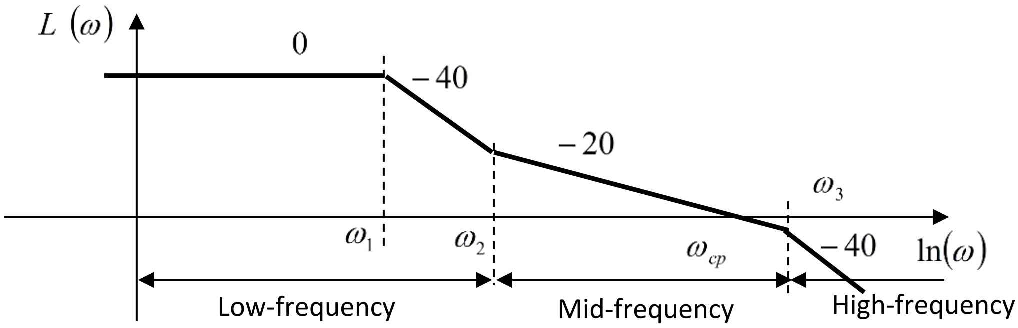

Fig. 5.8. Desired LgGFC

The desired LgGFC is built on the basis of requirements (specified quality indexes) to the system. LgGFC low-frequency asymptote of an open-loop system characterizes static properties. LgGFC mid-frequency asymptote and its conjugation with low-frequency characterize dynamic properties of the system are stability and quality indexes of transient characteristics. The desired LgGFC high-frequency asymptote has a meaningless impact on properties of the system. That's why it should be selected in that way, in order the corrective device to be possibly simple. This can be achieved by repeating inclines of LgGFC high-frequency asymptote of the original system, i.e. desired LgGFC $L_*(\omega)$ in this part goes parallel to the specified one.

Building the desired LgGFC of the mid-frequency asymptote is started from selecting cutoff frequency ω_{cp}. For this V. V. Solodovnikov's (В. В. Солодовников) nomographic chart is used [2]. The chart defines dependence of overcontrol and control time from maximum P_{max} of real frequency characteristics of a closed-loop system (Fig. 5.9).

Nomographic chart is used in the following way. By set value of overcontrol σ the value P_{max} is defined. Then by P_{max} the ratio of t_p and ω_{cp} is defined, having defined the value of parameter "*c*" by nomographic chart (Fig. 5.9).

For instance, suppose specified overcontrol 30%, and speeding of the system is t_p = 0.6 sec. Then, as you can see in Fig. 5.9, by σ = 30% we define P_{max} = 1.27 and after that we define c = 3,5. Consequently,

$$t_p = \frac{c\pi}{\omega_{cp}} = \frac{3{,}5\pi}{\omega_{cp}}$$

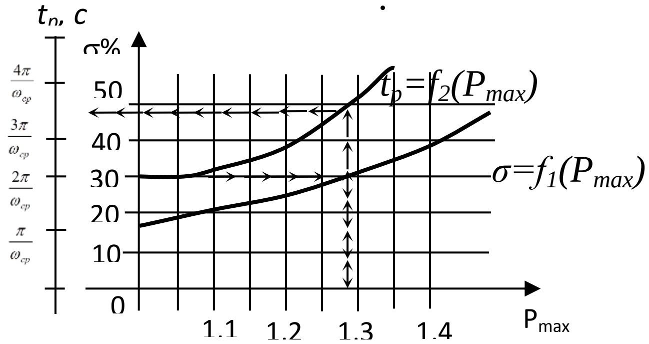

Fig. 5.9. Fragment of Solodovnikov's nomographic chart

Hence, $\omega_{cp} = \frac{3,5\pi}{t_p}$.

So, cutoff frequency of the desired LgGFC , in which reaction time will not exceed the specified value is calculated. The more cutoff frequency ω_{cp}, the less reaction time, and consequently, the system are more speeding.

The desired LgGFC mid-frequency asymptote is drawn through the point ω_{cp} with an incline – *20 db/dec*. If the incline is much more it is difficult to provide the required stability factor and admissible overcontrol.

The length of mid-frequency asymptote is set, sourcing from the required stability factor. For this reason also its conjugation is selected with a low-frequency asymptote. Except this, conjugating asymptote should be selected in such a way that characteristics $L_*(\omega)$ to be as less as possible differed from LgGFC of the original system and the corrective device was as simple as possible